



Top Threshold Physics

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Running a future Linear Collider at the top pair threshold allows for precise measurements of the mass, the widths and the couplings of the top quark. I give a nontechnical review on recent theoretical developments and the theory status in top threshold physics concerning QCD corrections and top quark finite lifetime and electroweak effects. I also discuss threshold physics in the context of measurements of the top Yukawa coupling from $e^+e^- \rightarrow t\bar{t}H$ and of squark pair production.

International Workshop on Top Quark Physics
January 12-15, 2006
Coimbra, Portugal

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1. Introduction

The measurement of the total $t\bar{t}$ cross section constitutes a major part of the top physics program at a future e^+e^- Linear Collider (LC). From the location of the rise of the cross section measurements of the top quark mass will be gained, while from the shape and the normalization one can extract the top quark width and get information on the top Yukawa coupling and the strong coupling. I will begin this talk with some comments on these measurement to provide an understanding of the requirements that are imposed on the theoretical predictions.

With a luminosity of a few times $10^{34}/(cm^2s)$ experimental simulation assume a total luminosity of several $100 fb^{-1}$ spent for a full threshold scan. For 10 scan points this corresponds to about $10^4 t\bar{t}$ pairs for c.m. energies where the cross section reaches the $1 pb$ level. To determine the observed experimental cross section $\sigma_{t\bar{t}}^{obs}$ one needs to have a very good knowledge on the luminosity spectrum $\mathcal{L}(x)$ which accounts for the machine-dependent beam energy spread, the effects of beamstrahlung and initial state radiation [1, 2],

$$\sigma_{t\bar{t}}^{obs}(\sqrt{s}) = \int_0^1 \mathcal{L}(x) \sigma_{t\bar{t}}^0(x^2 \sqrt{s}), \quad (1.1)$$

where $\sigma_{t\bar{t}}^0$ is the "partonic" cross section without initial state beam effects. The luminosity spectrum leads to a smearing of the partonic cross section and to a reduction of the observed cross section (see Fig. 1).

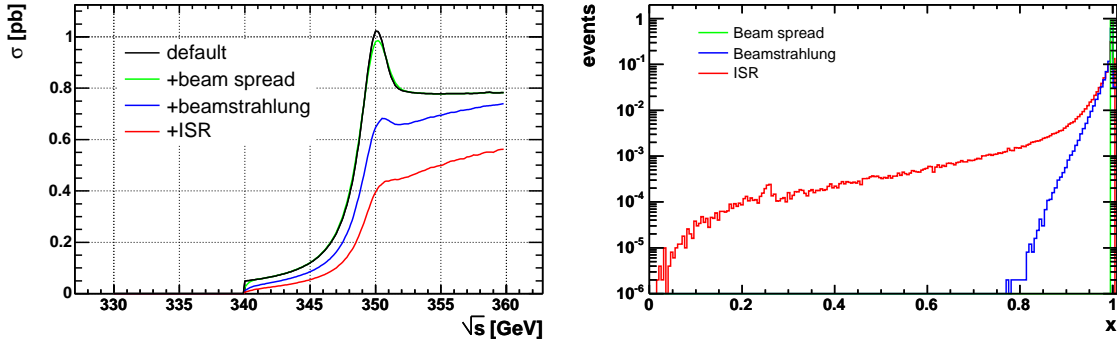


Figure 1: Left: Smearing of the "partonic" $t\bar{t}$ cross section by beam effects and initial state radiation. Right panel: Simulation of beam spread, beamstrahlung and ISR as distributions of $x = \sqrt{s}/\sqrt{s_0}$ (where $\sqrt{s_0}$ is the nominal c.m. energy of the machine). The figures are from Refs. [1, 3]

The c.m. energy where the cross section rises is strongly related to twice the top quark mass. The experimental statistical uncertainty in top quark mass measurements is around 20 MeV [4] and there is also an effect from uncertainties in the knowledge of $\mathcal{L}(x)$ which affects the top quark mass by probably less than 50 MeV [3]. In contrast to the mass reconstruction method that is traditionally used at the hadron colliders there is a very good knowledge on the intrinsic theoretical uncertainties in this type of top quark mass measurement. This is because the cross section line-shape can be computed precisely with perturbative methods. One can rely on perturbative methods because the rather large top quark width $\Gamma_t \approx 1.5$ GeV suppresses non-perturbative effects and prevents the formation of toponium bound states. So the lineshape can be computed as a function

of the Lagrangian top quark mass in any given scheme without ambiguities. The facts that we are considering a $t\bar{t}$ color singlet state and that one just needs to count $t\bar{t}$ event (in the experimental measurement) simplifies the task a lot. From NNLL order QCD computations and from general arguments based on studies of QCD perturbation theory at high orders [5] it is known that the best perturbative stability in the c.m. energy where $\sigma_{t\bar{t}}^0$ rises is obtained in so-called top threshold mass schemes [6] such as the 1S mass [7, 8], which I will for the rest of this presentation. This means that the top quark mass that is measured from the scan is a top threshold mass, such as the 1S mass. An important issue here is that threshold masses can be related reliably to e.g. the $\overline{\text{MS}}$ mass (see e.g. Ref. [8]) that is frequently used for new physics studies or electroweak precision observables. The top quark pole mass is known to lead to a much worse higher order behavior [6], but it never becomes relevant in these considerations. Simulations have shown that the 1S mass can be determined with theoretical uncertainties of about 100 MeV [9].

The top quark couplings and its total width can be determined from the normalization of the cross section and the details of the line-shape form. The strong coupling and the Yukawa coupling affect the attraction of the $t\bar{t}$ pair and determine the normalization. The top width determines the sharpness of the peak in $\sigma_{t\bar{t}}^0$. For 300 fb^{-1} distributed over 10 scan points the experimental errors are smaller than 50 MeV for the top width and at the level 0.001 for $\alpha_s(M_Z)$ [4]. If the Higgs is close to the present lower experimental bound, its Yukawa coupling to the top can be measured with around 35% precision. To achieve comparable theoretical errors the normalization and the line-shape form need to be known with a precision of better than 3%. As we will see below there is still some work to be investigated to reach this goal.

The physics at the top threshold involves a number of nontrivial theoretical issues related to the non-relativistic dynamics and the finite top quark lifetime that need to be addressed all at the same time. Gluon exchange leads to singular terms $\propto (\alpha_s/v)^n$ and $\propto (\alpha_s \ln v)^n$ in n -loop perturbation theory where $v \ll 1$ is the top velocity. The singularities enforce the parametric (power) counting $v \sim \alpha_s \ll 1$, i.e. one needs to expand simultaneously in α_s and v , and the use of an effective field theory (EFT) to sum them up to all orders in α_s . Due to the large top quark lifetime these computations can be carried out perturbatively based on the counting just mentioned. Interestingly the top width is approximately equal to the typical top kinetic energy $\Gamma_t \sim m_t \alpha \sim E_{\text{kin}} \sim m_t \alpha_s^2$, so the effects of the top lifetime cannot be treated as a perturbation and need to be implemented systematically starting from the LL approximation. Due to the relation between width and kinetic energy the combined expansion is based on the parametric counting

$$v \sim \alpha_s \sim \alpha^{1/2} \ll 1. \quad (1.2)$$

In the following sections I discuss the status of the theoretical predictions for the total cross section $\sigma_{t\bar{t}}^0$ concerning QCD (Sec. 2) and finite lifetime and electroweak effects (Sec. 3). In Secs. 4 and 5 I discuss applications of these theoretical tools to Yukawa coupling measurements from $e^+e^- \rightarrow t\bar{t}H$ and squark pair production.

2. QCD Effects

Schematically the perturbative expansion and summations for the cross section have the form

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_{k,i} \left(\frac{\alpha_s}{v} \right)^k (\alpha_s \ln v)^i \{ 1 \text{ (LL)}, \alpha_s, v \text{ (NLL)}, \alpha_s^2, \alpha_s v, v^2 \text{ (NNLL)} \}. \quad (2.1)$$

where the terms α_s/v and $\alpha_s \ln v$ are counted of order 1. The singular terms originate from ratios of the physical scales m_t (hard), $\mathbf{p}_t \sim m_t v$ (soft) and $E_t \sim m_t v^2$ (ultrasoft). The summations are achieved systematically in the various orders of approximation by construction of a low energy EFT, generically called nonrelativistic QCD (NRQCD), that describes correctly the nonrelativistic fluctuations of full QCD for the kinematic situation of the top quarks close to threshold. A number of different versions of NRQCD exist [10, 11, 12, 13], each of which aiming (in principle) on applications in different physical situations. The EFT vNRQCD (“velocity”NRQCD) [12, 14] has been designed for predictions at the $t\bar{t}$ threshold in the scheme (2.1). It treats the case $m_t \gg \mathbf{p}_t \gg E_t > \Lambda_{\text{QCD}}$, i.e. all physical scales are perturbative, but also has the correlation $E_t = \mathbf{p}_t^2/m_t$ built in at the field theoretic level. The latter is important to achieve the correct summation of the logarithmic $\ln v$ terms by renormalization group evolution. The EFT consists of a Lagrangian with local operators made from top and gluon fields that describe the quantum fluctuations that are resonant at the nonrelativistic scales \mathbf{p}_t and E_t . High energy fluctuations that occur in EFT loop diagrams and off-shell fluctuations are accounted for in the EFT renormalization procedure and by matching the EFT coefficients to the full theory at the hard scale m_t . This fixes the matching (initial) conditions and the renormalization group running of the coefficients. The large logarithms $\ln v$ are summed by evolving the coefficient to the low-energy scale such that all large logs disappear from the EFT matrix elements. At LL order the EFT Lagrangian relevant for $e^+ e^- \rightarrow t\bar{t}$ at threshold has the simple form

$$\begin{aligned} \mathcal{L}(x) = & \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger \left\{ iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3} + \frac{i}{2}\Gamma_t \left(1 - \frac{\mathbf{p}^2}{2m_t^2} \right) - \delta m_t(v) \right\} \psi_{\mathbf{p}} + (\psi_{\mathbf{p}} \rightarrow \chi_{\mathbf{p}}) \\ & - \sum_{\mathbf{p}, \mathbf{p}'} V(\mathbf{p}, \mathbf{p}') \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}, \\ V(\mathbf{p}, \mathbf{p}') = & \frac{\mathcal{V}_c(v)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(v)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(v)(\mathbf{p}^2 + \mathbf{p}'^2)}{2m_t^2 \mathbf{k}^2} + \frac{\mathcal{V}_2(v)}{m_t^2} + \frac{\mathcal{V}_s(v)}{m_t^2} \mathbf{S}^2, \quad (\mathbf{k} = \mathbf{p}' - \mathbf{p}) \end{aligned} \quad (2.2)$$

where $\psi_{\mathbf{p}}$ and $\chi_{\mathbf{p}}$ destroy top and antitop quarks with momentum \mathbf{p} and D^μ is the covariant derivative with respect to ultrasoft gluons; the term $V(\mathbf{p}, \mathbf{p}')$ contains the Coulomb potential. I have also shown a few terms that come in at NNLL. All couplings are functions of the dimension-less renormalization group scaling parameter v . At $v = 1$ (hard scale m_t) the coefficients are determined from the matching procedure and at $v \sim v \sim \alpha_s$ the EFT matrix elements are computed. The scaling from $v = 1$ to α_s sums all logs of ratios of the scales m_t , \mathbf{p}_t and E_t and also accounts for the correlation of \mathbf{p}_t and E_t . The coefficient δm_t is determined (unambiguously order by order) by the mass definition that is used. The term $\psi_{\mathbf{p}}^\dagger i\Gamma_t \psi_{\mathbf{p}}$ accounts for the top decay at LL order, which I will discuss in more detail in the next section. The equation of motion for the $t\bar{t}$ system obtained from Eq. (2.2) is a Schrödinger equation. At NNLL order it has, in configuration space, the form

$$\left(-\frac{\nabla^2}{m_t^2} - \frac{\nabla^4}{4m_t^3} + V(\mathbf{r}) - (\sqrt{s} - 2m_t - 2\delta m_t(v) + i\Gamma_t) \right) G(\mathbf{r}, \mathbf{r}', \sqrt{s}, v) = \delta^{(3)}(\mathbf{r} - \mathbf{r}'), \quad (2.3)$$

where G is the Green function.¹ The Lagrangian does not describe $t\bar{t}$ production or annihilation. This is done by additional operators (external currents). The dominant operators, which describe $t\bar{t}$ production in a S-wave spin-triplet state are

$$\mathcal{O}_{V,A\mathbf{p}} = C_{V,A}(\nu) \left[\bar{e} \gamma_j (\gamma_5) e \right] \left[\psi_{\mathbf{p}}^\dagger \sigma_j (i\sigma_2) \chi_{-\mathbf{p}}^* \right]. \quad (2.4)$$

where the coefficients $C_{V,A}(\nu)$ describe the hard, nonresonant fluctuation that are involved in the $t\bar{t}$ production process. Using the optical theorem one can obtain the total cross section,

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} \sim \text{Im} \left[(C_V^2(\nu) + C_A^2(\nu)) G(0, 0, \sqrt{s}, \nu) \right], \quad (2.5)$$

where the zero-distance Green function describes the nonrelativistic dynamics of the $e^+e^- \rightarrow e^+e^-$ forward scattering amplitude with the $t\bar{t}$ pair being produced and annihilated at the origin. The zero-distance Green function is fully known at NNLL order (see e.g. Ref. [15] for details). The matching conditions $C_{V,A}(1)$ are known at NNLL order. The renormalization evolution of the coefficients, $f(\nu) = C_{V,A}(\nu)/C_{V,A}(1)$ is known at NLL order [14, 16]. At NNLL order the 3-loop non-mixing contributions are known for the function f [17], but the subleading evolution of the coefficients that go into the NLL evolution of f (mixing contributions) has not yet been determined. In Fig. 2 the predictions for R at LL (blue dotted lines), NLL (green dashed lines) and NNLL order without the unknown mixing corrections (red solid lines) are shown for $m_t = 175$ GeV, $\Gamma_t = 1.43$. We see that the NNLL corrections are substantial, and that the NNLL prediction has a much larger ν -dependence than the NLL order one. At present the QCD normalization uncertainty is around 6% [18]. This is far away from the 3% goal, but a final conclusion has to await the completion of the missing NNLL order corrections or even higher order computations. I personally believe that it will be the full NNLL order prediction that determines the final QCD uncertainty that can be achieved.

3. Finite Lifetime and Electroweak Effects

Until now most effort in the literature went into the analysis and determination of QCD effects. Electroweak and in particular finite top quark lifetime corrections have received much less attention beyond the LL order level. In fact not even the full set of NLL order corrections (based on the counting in Eq. (1.2)) are known for the total cross section. For the treatment of finite lifetime corrections no fully general method, that can at the same time handle all realistic cases and observables, exists. In a given set of (reasonable) approximations and for specific observables, however, a systematic and consistent approach can be developed. In Ref. [19] the EFT approach developed for the QCD effects was extended to determine the finite topquark lifetime corrections to the total cross section for the powercounting (1.2) in the approximation of a stable W boson. The approach is very similar to the theory of light propagation in an absorptive medium, where the effects of absorption can be encoded in complex contributions to the coefficients of the vacuum theory as long as one does not want to address microscopic details of the absorption processes.

¹The LL zero-distance Green function in dimensional regularization has the simple analytic form $G^0(0, 0, \sqrt{s}, \nu) = \frac{m_t^2}{4\pi} \left\{ i\nu - a \left[\ln\left(\frac{-i\nu}{\nu}\right) - \frac{1}{2} + \ln 2 + \gamma_E + \psi\left(1 - \frac{ia}{2\nu}\right) \right] \right\} + \frac{m_t^2 a}{4\pi} \frac{1}{4\varepsilon}$, with $a = C_F \alpha_s(m_t \nu)$ and $\nu = ((\sqrt{s} - 2m_t - 2\delta m_t(\nu) + i\Gamma_t)/m_t)^{1/2}$.

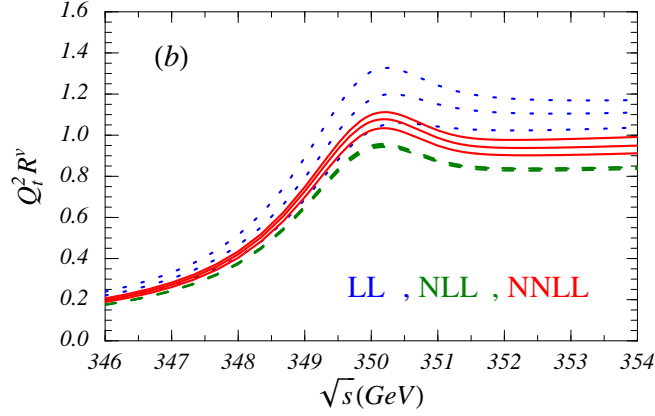


Figure 2: Predictions for R in renormalization group improved perturbation theory at LL (dotted lines), NLL (dashed lines) and NNLL (solid lines) order. For each order curves are plotted for $v = 0.15, 0.20$, and 0.3 . The effects of the luminosity spectrum are not included. [18]

Electroweak effects can be categorized into three classes:

- (a) "Hard" electroweak effects: This class includes hard, point-like electroweak effects related e.g. to the $t\bar{t}$ production mechanism by virtual photon and Z exchange, or corrections to various matching conditions of the EFT. In general these corrections are modifications of the hard QCD matching conditions of the EFT operators. They can be determined by standard methods by matching at the top quark complex pole, and are real numbers.
- (b) Electromagnetic effects: They are relevant for the luminosity spectrum of the e^+e^- initial state. The other low- and high energy properties of photon interactions of the $t\bar{t}$ pair and its decay products are similar to the gluonic corrections and can be incorporated in the same way into the EFT, but their effects are in general of higher order (see Eq. (1.2)). At NNLL order a coherent treatment of all electromagnetic effects is required.
- (c) Effects related to the finite top quark lifetime: Apart from the top decay (into Wb for the Standard Model) this class also includes interference contributions with processes having the same $W^+W^-b\bar{b}$ final state but only one or even no top quark at intermediate stages. It also accounts for interactions involving the top decay products (sometimes called "nonfactorizable" effects). In Ref. [19] it was shown that, as long as the top decay is treated inclusively, finite lifetime effects can be incorporated into the EFT matching conditions by imaginary contribution that are determined from those (and only those) cuts in electroweak matching corrections that are related to the top decay. As for class (a) the matching procedure is carried out at the top quark complex pole (although this issue does not become relevant up to NNLL order). One might say that the top decay is integrated out, although this notion can be misleading. This renders the EFT non-Hermitian, but unitarity is still preserved due to the Hermiticity of the full electroweak theory. Gauge invariance is maintained at all times (as long as only gauge invariant sets of operators are used in the EFT).

Let me now discuss the status concerning the electroweak effects for the total cross section coming from the three classes and using the power counting (1.2): At **LL order** all contributions are

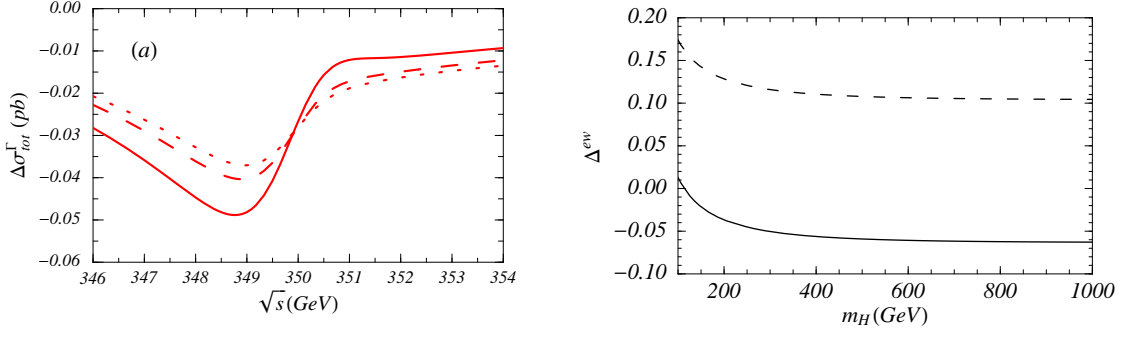


Figure 3: Left panel: Corrections to the total cross section from NNLL time-dilatation and interference effects and NLL summation of phase space logarithms [19]. Right panel: Relative normalization corrections to the total cross section from NNLL hard electroweak corrections for $\alpha = 1/137$ (dashed line) and for a scheme with an electromagnetic coupling at the m_t scale, $\alpha^{n_f=8}(\mu = m_t) = 125.9$ [23].

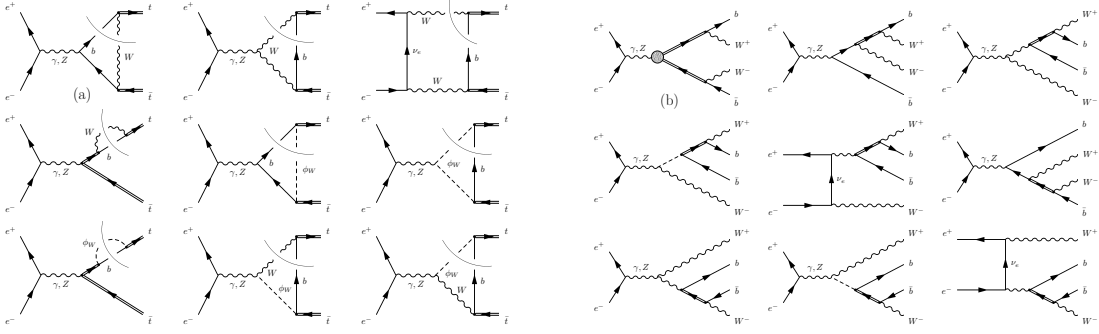


Figure 4: Left panel: Full theory diagrams in Feynman gauge needed to determine the electroweak absorptive parts in the Wilson coefficients $C_{V/A}$ related to the physical bW^+ and $\bar{b}W^-$ intermediate states. Only the bW^+ cut is drawn explicitly. Right panel: Full theory diagrams describing the process $e^+e^- \rightarrow bW^+ \bar{b}W^-$ with one or two intermediate top or antitop quark propagators. The circle in the first diagram represents the QCD form factors for the $t\bar{t}$ vector/axial-vector currents.

known. There are the tree level matching conditions $C_{V/A}(1)$ to the coefficients of the $t\bar{t}$ production and annihilation operators which describe the intermediate virtual photon and Z exchange in e^+e^- annihilation (class(a)), the luminosity spectrum in Eq. (1.1) (class (b)) and the imaginary width term $\psi_p^\dagger i\Gamma_t \psi_p$ in the kinetic terms of the EFT Lagrangian in Eq. (2.2). The width term arises from the cut in the full theory top quark electroweak selfenergy diagrams in the matching procedure. At **NNLL order** there are no corrections in class (a), because hard electroweak matching corrections beyond LL order can only contribute at NNLL order according to (1.2). Moreover the NNLL class (a,c) matching corrections to the Coulomb potential \mathcal{V}_c and the dominant top- A^0 gluon interactions vanish due to gauge invariance. In class (b) there is an additional QED contribution to the Coulomb potential from the exchange of a Coulomb photon (see [20]). A similar correction does not exist at NNLL order. In class (c) there are $\mathcal{O}(\alpha_s)$ QCD corrections to the top width Γ_t [21] and phase space corrections that are explained below. The statement I made on the class (a) corrections implies a non-trivial cancellation of effects that involve real and virtual ultrasoft gluon exchange among the top quarks and its decay products that appears non-trivial from the diagrammatic point of view [22].

At **NNLL order**, in class (a) there are one-loop electroweak corrections in class (a) to the matching conditions $C_{V,A}(1)$. They were determined in Refs [24, 25] and reanalyzed with corrections recently in Ref. [23]. They lead to an energy-independent normalization correction to the total cross section that is shown in Fig. 3 as a function of the Higgs mass. The determination of NNLL order electromagnetic corrections has not been attempted until now. The class (c) finite lifetime corrections were analyzed in Ref. [19]. They include the $\mathcal{O}(\alpha_s^2)$ QCD corrections to Γ_t [26], the time-dilatation correction $\propto \Gamma_t \mathbf{p}^2/m_t$ (see Eq. (2.2)) and the one-loop imaginary contributions to $C_{V,A}(1)$ from cuts related to the top decay in the one-loop corrections to $e^+e^- \rightarrow t\bar{t}$ (see Fig. 4a). Due to unitarity the imaginary contribution have the same sign for the $t\bar{t}$ production and the annihilation operators and lead to a energy-dependent correction to the total cross section (2.5). It was shown in Ref. [19] that these corrections account for the interference of the dominant amplitude $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}W^+W^-$ with amplitudes having the same final state, but only one top or antitop as intermediate lines (Fig. 4b). These interference corrections lead to a new type of UV-divergence that can be seen from Eq. (2.5) and the UV-divergence in the real part of the zero-distance Green function, as can be seen in footnote 1. The divergence is related to the Breit-Wigner type EFT top quark propagator (see Eq. (2.2) and the fact that the EFT phase space is infinite. This is because the EFT is based on an expansion around the top mass shell region, which in this context means taking $m_t \rightarrow \infty$. The UV-divergence has to be renormalized by imaginary counterterms of $(e^+e^-)(e^+e^-)$ forward scattering operators [19] which also have to be added to the RHS of Eq. (2.5). The renormalization group evolution of these operators is a NLL order effect (just like the LL running of α_s is determined from NLL one-loop diagrams) and was determined in Ref. [19]. It sums phase space logarithms $\propto \Gamma_t(\alpha_s \ln v)^n$ to all orders in α_s . The impact of the sum of time-dilatation and interference corrections and the NLL order phase space logarithms is shown in Fig. 3a. The corrections are energy-dependent and particularly large where the cross section is small and the top quarks are further off-shell. They leads to a shift of the peak position in $\sigma_{t\bar{t}}^0$ by 30 to 50 MeV. The results in Fig. 3 demonstrate that the NNLL electroweak corrections are comparable to the NNLL QCD effects and need to be determined to reach the 3% goal discussed at the beginning of this talk.

4. Threshold Physics and $e^+e^- \rightarrow t\bar{t}H$

It is one of the major tasks of the future Linear Collider to unravel details of the mechanism of electroweak symmetry breaking. One of the crucial measurements is the (as much as possible) model-independent determination of the top Yukawa coupling λ_t which in the Standard Model is related to the top mass and the vacuum expectation value. At the e^+e^- Linear Collider the top quark Yukawa coupling can be measured from top quark pair production associated with a Higgs boson, $e^+e^- \rightarrow t\bar{t}H$, since the process is dominated by the amplitude describing Higgs radiation off the $t\bar{t}$ pair. This process is particularly suited for a light Higgs boson since the cross section can then reach the 1-2 fb level and measurements of λ_t (close to the Standard Model value) with relative errors of a few percent are expected [27]. With this motivation one-loop QCD [28, 29] and also electroweak corrections [30, 31, 32] were determined. There is, however, a region in the phase space where the Higgs energy is large and the $t\bar{t}$ dynamics is nonrelativistic. For large Higgs energies the $t\bar{t}$ pair is forced to become collinear and to move opposite to the Higgs direction in order to achieve the large total momentum necessary to balance the large Higgs momentum (Fig. 5). In this kinematic

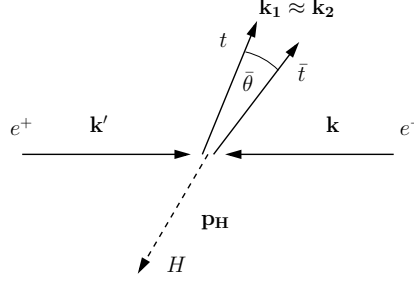


Figure 5: Typical constellation of momenta for the process $e^+e^- \rightarrow t\bar{t}H$ in the large Higgs energy endpoint region.

region the $t\bar{t}$ invariant mass $Q_{t\bar{t}}$ is close to $2m_t$, i.e. the $t\bar{t}$ pair is nonrelativistic in its c.m. frame. For a relatively light Higgs below the W^+W^- threshold the Higgs width is only at the level of several MeV and it is therefore possible to neglect gluon interactions between the top quarks and the Higgs decay products. So the QCD dynamics of the $t\bar{t}H$ system in the large Higgs energy endpoint region is very similar to the physics at the $t\bar{t}$ threshold discussed in the previous sections. In particular, the usual loop QCD perturbation theory breaks down due to $(\alpha_s/v)^n$ and $(\alpha_s \ln v)^n$ singularities and an EFT treatment is required. In Ref. [33] a NLL order QCD factorization formula in close analogy to Eq. (2.5) was derived for the Higgs energy spectrum in the large Higgs energy endpoint region using the formalism developed for the $e^+e^- \rightarrow t\bar{t}$ threshold. It has the (simplified) form

$$\frac{d\sigma}{dE_H}(E_H \approx E_H^{\max}) = h(\sqrt{s}, m_t, m_H) \text{Im} [C^2(v) G(0, 0, Q_{t\bar{t}}(E_H), v)] , \quad (4.1)$$

where the constant h accounts for the hard electroweak effects (where arguments such as the Z mass and the electroweak couplings are not written) and $C(v)$ for the hard QCD corrections of the $t\bar{t}H$ production mechanism. For c.m. energies above 500 GeV the summation of the terms singular in v leads to corrections to the known $\mathcal{O}(\alpha_s)$ one-loop predictions for the total cross section since for large c.m. energies only a part of the phase space is dominated by the nonrelativistic $t\bar{t}$ dynamics. Below 500 GeV, however, the energy available during the first phase of a LC program based on the cold technology, the maximal possible relative top velocity is so small that the full phase space is nonrelativistic, i.e. the physics at the large Higgs energy endpoint governs the full phase space. This makes the loop expansion in powers of α_s unreliable and the nonrelativistic expansion based on Eq. (1.2) has to be applied. In Ref. [34] the factorization formula (4.1) was extended to also account for the correct physical behavior at the low Higgs energy endpoint $E_H = m_H$ and the resulting NLL order QCD predictions were analyzed for $\sqrt{s} \leq 500$ GeV. (The NLL order electroweak and finite lifetime corrections are still unknown.) In Fig. 6a the Higgs energy spectrum at LL (red dotted lines) and NLL order (red solid lines) are shown for $\sqrt{s} = 490$ GeV and $m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $m_H = 120$ GeV and $v = 0.1, 0.2, 0.4$. As a comparison also the tree level (blue dashed line) and $\mathcal{O}(\alpha_s)$ (blue solid line) predictions (for $\Gamma_t = 0$) are shown. In Fig. 6b the total cross section is shown as a function of the c.m. energy for the same choice of the other parameters as in Fig. 6a at NLL order (red lines) and tree level (blue lines). The dashed lines are for unpolarized e^+e^- beams, $(P_{e^+}, P_{e^-}) = (0, 0)$, and the solid lines for $(P_{e^+}, P_{e^-}) = (0.8, -0.6)$. We see that the

NLL predictions are substantially larger than the tree level ones, by roughly a factor of two. The cross section can be even further enhanced when polarized e^+e^- beams are used. Since the past experimental simulation analyses for top Yukawa coupling measurements at c.m. energies up to 500 GeV were based on tree level theory predictions and unpolarized beams², it can be expected that the NLL predictions will have a substantial (positive) impact on the prospects for top Yukawa coupling measurements at these energies.

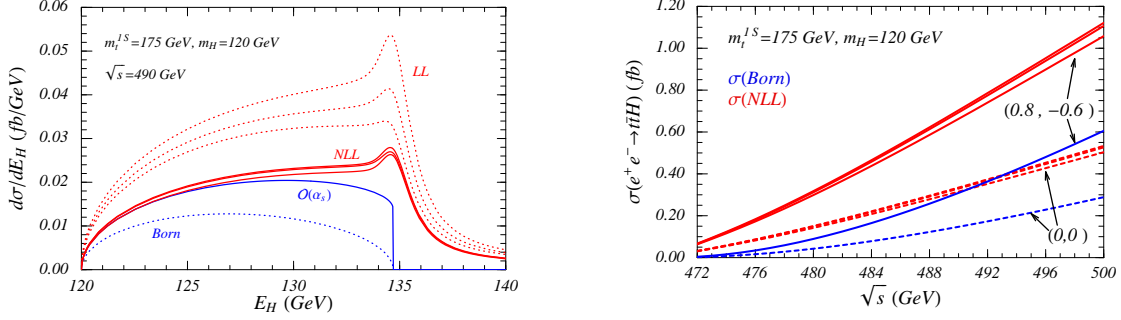


Figure 6: Left panel: Higgs energy spectrum for the process $e^+e^- \rightarrow t\bar{t}H$ at $\sqrt{s} = 490$ GeV for $m_t = 175$ GeV, $m_H = 120$ GeV, $\Gamma_t = 1.43$ GeV at NLL (red solid lines) and LL order (red dotted lines) for $v = 0.1, 0.2, 0.4$. Also displayed are the tree level and $\mathcal{O}(\alpha_s)$ results for stable top quarks. Right panel: Total cross section $\sigma(e^+e^- \rightarrow t\bar{t}H)$ as a function of the c.m. energy for the same parameter set at tree level (blue lines) and NLL order (red lines) for unpolarized (dotted lines) and polarized e^+e^- beams with $(P_{e^+}, P_{e^-}) = (0.8, -0.6)$ (solid lines).

5. Threshold Production of Squark Pairs

Many models of supersymmetry breaking predict that at least one of the supersymmetric partners of the top quark is sufficiently light such that stop-antistop pair production is possible at a future Linear Collider running at c.m. energies up to 1 TeV. In such a scenario threshold measurements in analogy to the $t\bar{t}$ threshold will be possible [36]. An important difference to the $t\bar{t}$ case is, however, that squark pairs are predominantly produced in a P-wave in e^+e^- annihilation such that the rise of the total cross section at the threshold is v^2 -suppressed and substantially slower than for $t\bar{t}$ production. (For $\gamma\gamma$ collisions squark pairs are produced predominantly in an S-wave configuration.) Up to now there have not even been consistent LL order predictions for this P-wave process because, here, the top quark finite lifetime issues that became relevant at NLL and NNLL order for S-wave production come in already at LL order. In Ref. [37] a first step toward a systematic treatment of the squark pair threshold was done by construction the scalar version of vNRQCD relevant for the description of the NLL and NNLL QCD effects.

Acknowledgements

I would like to thank my collaborators C. Farrell, C. Rei er and P. Ruiz-Femenia for their excellent work and enthusiasm. I also thank the organizers of International Workshop on Top Quark Physics for the pleasant and inspiring atmosphere during the conference.

²In the experimental analysis of Ref. [35] $(\delta\lambda_t/\lambda_t)^{\text{ex}} = 25\%$ was obtained.

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